INTRODUCING THE **UNITS 3&4 EDROLO FURTHER MATHS** TEXTBOOKS

Edrolo

VCE FURTHER MATHEMATICS

Units 3&4

- **VCAA-style questions** based on a thorough analysis of past exams
- **Exemplar answers** with a **video solution** for every multiple lesson question
- **Step-by-step calculator instructions** for both TI-nspire and CASIO Classpad
- **Concise theory** covering the core knowledge required within the scope of the VCAA Study Design
- **Edrolo Theory Master videos aligned** directly to textbook chapters
- **Full lessons dedicated to difficult concepts** deep dive into areas of the Study Design students find most challenging:
  - Decoding VCAA questions to identify the skills required
  - Utilising calculator tools and functions

3RD EDITION

AT THE **BEGINNING OF EACH LESSON YOU WILL FIND:**

Key Knowledge dot-points from the study design provide explicit links to the syllabus.

Theory summaries provide concise explanations and examples of the relevant theory for each Key knowledge dot point.

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8.E USING NETWORKS TO MODEL FLOW PROBLEMS

Key knowledge

- use of networks to model flow problems: capacity, sinks and sources
- solution of small-scale network flow problems by inspection and the use of the 'maximum-flow minimum-cut' theorem to aid the solution of larger scale problems

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.61; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

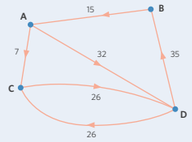
Theory summary

Directed graph

Directed graphs, or digraphs, are networks containing arrows on each edge. These arrows show directional information between vertices.

Example

The network below represents the roads connecting four towns. The arrow on the edge from town A to town D shows that it is a one way road. That is, people can travel directly from town A to town D, but not from town D to town A. The two edges between towns C and D show that traffic is allowed in both directions.



Flow problems

When networks only allow for movement, or flow, in one direction, they start at one vertex (the source) and end at another vertex (the sink). Flow problems are based around the movement of flow from source to sink.

Example

The network below shows the flow from source to sink (left to right).

Maximum flow

The maximum flow is equal to the minimum capacity through a network.

Example

The start of a river system has the capacity to allow 7 kilolitres of water to flow through it each second. The river then widens and the capacity of the next section of river is 13 kilolitres per second. However, since the first section of river is smaller, only 7 kL will be allowed into the river system each second. Therefore, the full capacity of the second section will not be used.

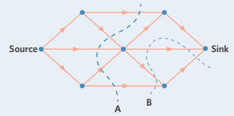
The maximum flow is equal to the minimum capacity. The maximum flow of the river system is 7 kL/sec.

Cut

A cut is a line passing through a network that cuts the network in two. It must separate the source from the sink and completely stop the network flow.

Example

On the network below, line A is a cut because it completely prevents any flow from the source to the sink. Line B is not a cut because it does not prevent all flow from source to sink.



676 **MODULES:** GRAPHS AND RELATIONS

Stationary point of inflection

A cubic of the form $y = kx^3$ levels out and becomes horizontal at the origin. This is called a stationary point of inflection.

Example

Both of the examples on the last page have stationary points of inflection at (0,0).

WORKED EXAMPLES

WORKED EXAMPLE 1 (2 MARKS)

Plotting a graph within a specificed domain using CAS

The groundskeeper at Hogwarts School of Witchcraft and Wizardry wishes to redesign the posts on the quidditch field as the bludger balls made extreme indents in the old versions. After hours of deliberation, the groundskeeper decides that the design will follow the curve given by the equation $y = \frac{16}{3}x^2$ where $-1 \leq x \leq 1$. Plot the graph to give the design of the quidditch post.

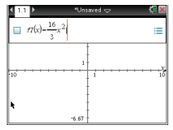
Solution: Using TI-Nspire

Step 1 Open a new page by pressing ctrl + doc and select "Add Graphs".

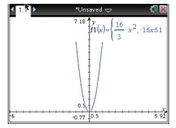
Specifying the domain

The domain of a graph can be restricted by entering the symbol "|" followed by the specific domain given.

Step 2 Type the equation $y = \frac{16}{3}x^2$ into the tab bar followed by "$| -1 \leq x \leq 1$".



To specify the domain, press the Ctrl button and then the "=" button directly underneath. Select |.



Marks	Key points
1 mark	Correct domain
1 mark	Correct graph shape

Solution: Using Casio Classpad

Step 1 Press menu.

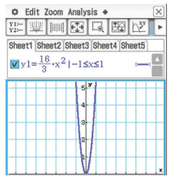
Step 2 In y1 enter "$y=\frac{16}{3}x^2$|".

Specifying the domain

The domain of a graph can be restricted by entering the symbol "|" (found in the keyboard under the heading "math3") followed by the specific domain.

Step 3 Enter "|", followed by the domain "$-1 \leq x \leq 1$" and press "EXE".

Step 4 Press to graph. Then select "Zoom → Auto" to view the whole curve.



Marks	Key points
1 mark	Correct domain
1 mark	Correct graph shape

Worked examples guide students through the analysis of a question, the steps required, and a thorough work-through of how to reach the answer.

Step-by-step calculator instructions for both Casio Classpad and the TI-Nspire.

QUESTIONS & SOLUTIONS

468 **MODULES:** NETWORKS AND DECISION MATHS

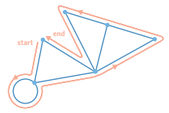
QUESTIONS See full solutions at **edrolo.com.au**

Identifying types of walks

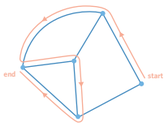
QUESTION 1 (6 MARKS) DIFFICULTY: Skill

Determine if the walk in each of the following graphs is a trail, path, circuit, cycle or just a walk.

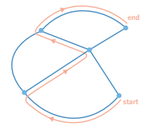
a) (1 MARK)



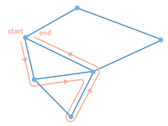
b) (1 MARK)



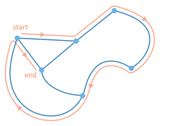
c) (1 MARK)



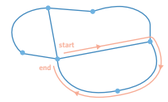
d) (1 MARK)



e) (1 MARK)



f) (1 MARK)



Eulerian trails and circuits

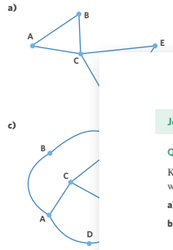
QUESTION 2 (8 MARKS) DIFFICULTY: Skill

For each of the following graphs:

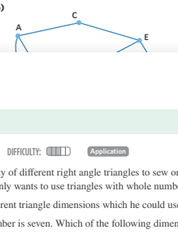
i. Identify whether an Eulerian trail or Eulerian circuit is possible. (1 MARK)

ii. Write down an Eulerian trail or circuit starting from vertex A, if possible. (1 MARK)

a)



b)



c)

9.A PYTHAGORAS' THEOREM 537

Joining it all together

QUESTION 10 (2 MARKS) DIFFICULTY: Application

Khamal needs to cut a variety of different right angle triangles to sew onto his dress for his fashion project. He isn't very good with measurement, and so only wants to use triangles with whole number lengths.

a) Come up with five different triangle dimensions which he could use to cut his triangles. (1 MARK)

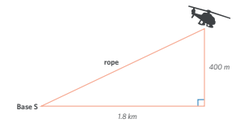
b) Khamal's favourite number is seven. Which of the following dimensions incorporating the number seven could he use for his project? (1 MARK)

A. $a = 7, b = 24, c = 25$ **B.** $a = 6, b = 6, c = 7$

C. $a = 5, b = 7, c = 11$ **D.** $a = 4, b = 7, c = 14$

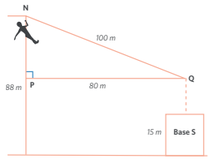
QUESTION 11 (7 MARKS) DIFFICULTY: Application

Amelia is trying to steal some secret documents from an enemy base, Base S. Her helicopter is hovering 400 m in the air. Amelia is going to jump out of the helicopter and walk 1.8 km west to the base. To aid a getaway, she is to be attached by a rope to the helicopter, which will be able to reach her all the way to the base.

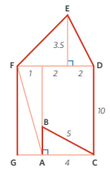


a) How long does this rope need to be? (1 MARK)

b) Amelia needs to zipline into the base. She stands on a platform 88 m above ground level, and using a 100 m long zipline aims to drop from Q onto the 15 m tall base 80 m away. How far does Amelia need to drop? Give your answer correct to the nearest metre. (2 MARKS)



c) Upon arrival Amelia sees that in order to reach the documents at G, she needs to enter the base at A and follow the bolded pathway in order to avoid guards. Correct to one decimal place, calculate the distance that Amelia must travel. All distances in the diagram are in metres. (4 MARKS)



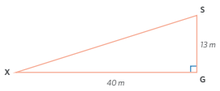
VCAA questions

QUESTION 12 (1 MARK) DIFFICULTY: VCAA

VCAA 2013 Exam 2 Module 2: Geometry and Trigonometry, Q1a

A spectator, *S*, in the grandstand of an athletics ground is 13 m vertically above point *G*. Competitor *X*, on the athletics ground, is at a horizontal distance of 40 m from *G*.

Find the distance, *SX*, correct to the nearest metre.



870 SOLUTIONS - CHAPTERS 9-10

Information we are given

- Oakland is located at latitude 38°N and longitude 122°W.
- The radius of Earth is 6400 km.

Using the theory

The radius of the small circle of the Earth at any latitude can be found using the formula $r = r_{com} \times \cos(\theta)$, where θ is the latitude of the small circle (N or S).

The latitude of Oakland, and hence, the latitude of the small circle in question is 38°, $\theta = 38$.

The radius of Earth is 6400 km; $r_{com} = 6400$.

Substitute $\theta = 38$ and $r_{com} = 6400$ into $r = r_{com} \times \cos(\theta)$.

$r = 6400 \times \cos(38) = 5043.268...$

∴ The radius of the small circle of Earth at Oakland's latitude, rounded to the nearest kilometre, is 5043 km.

Answer

5043 km

1 mark + Correct answer

13bii *Geometry and measurement*

Decoding the question

Keyword: 'small circle distance'

Relevant lesson: 9L

Information we are given

- Oakland is located at latitude 38°N and longitude 122°W.
- Athens is located at latitude 38°N and longitude 24°E.
- The radius of the small circle of Earth at Oakland's latitude is 5043 km.

Using the theory

Both Oakland and Athens have the same latitude: 38° N.

Hence, both cities are situated on the same small circle. This small circle is shown in 2D in the following diagram (as viewed from the North Pole).

The shortest small circle length between Oakland and Athens is equal to the minor arc length.

The length of an arc can be found using the formula $s = \frac{\theta\pi r}{180}$.

We found the radius of the small circle of Earth at latitude 38° in part b) to be 5043 km.

Hence, $r = 5043$.

The angle of the minor arc between Oakland and Athens on this small circle is equal to 122° + 24° = 146°.

Hence, $\theta = 146$.

Substitute $r = 5043$ and $\theta = 146$ into $s = \frac{\theta\pi r}{180}$.

$s = \frac{\pi \times 5043 \times 146}{180} = 12\,850.475...$

∴ The shortest small circle distance between Oakland and Athens, to the nearest 10 kilometres, is 12 850 km.

Note: Using the unrounded radius of the small circle from part b) yields the same answer.

Answer

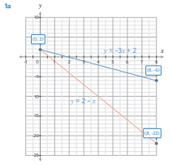
12 850 km

1 mark + Correct answer

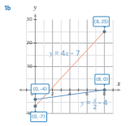
CHAPTER 10

10.A Straight line graphs

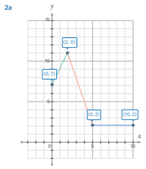
1a



1b



2a



SCAFFOLDED TEXTBOOK QUESTIONS:

- **Skills questions:** Both short answer and multiple-choice questions designed to consolidate understanding and execution of each particular skill.
- **Joining it all together** questions draw on multiple skills to provide frequent, valuable SAC & exam practice.
- **VCAA question:** Relevant questions from previous VCAA exams to test practical application of the theory in the lesson.
- **Questions from multiple lessons** are adapted from previous VCAA exams to test knowledge from content that was previously covered to reduce 're-leaning' in the lead up to exams. Students will also practice their decoding skills.

TEXTBOOK SOLUTIONS:

- **Worked solutions for multiple-lesson questions** guide students through 'Decoding the question', extracting key information and 'Using the theory'.
- **Video solutions for every textbook question in your Edrolo account,** including calculator instructions.

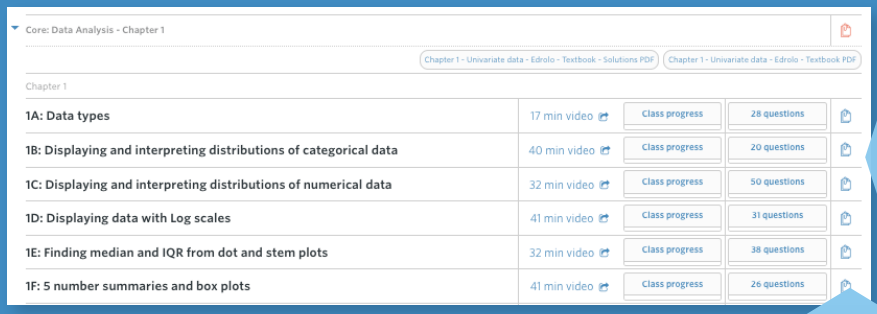
FOR **MORE INFO** SEE THE **TEXTBOOK TRAINING VIDEO** IN **YOUR EDROLO ACCOUNT**

NAVIGATING YOUR EDROLO TEXTBOOK **ONLINE**

PLANNING AHEAD

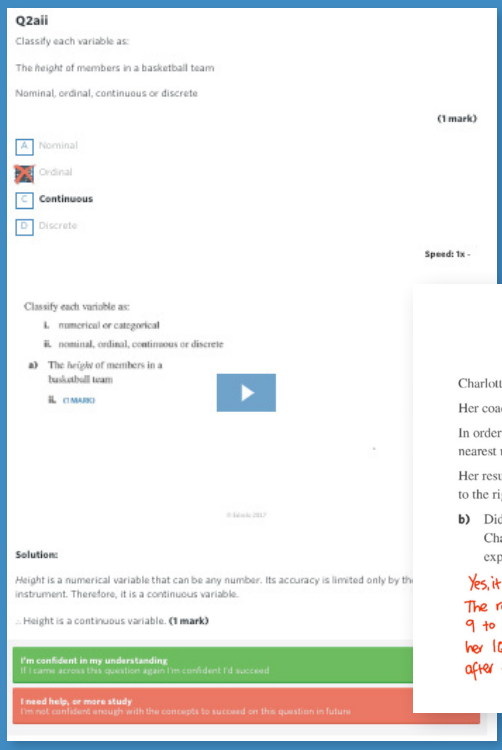
To assist with planning, there is an editable and downloadable **unit plan** available for your course.

ACCESSING THE TEXTBOOK PDFS, DIGITAL TEXTBOOK QUESTIONS AND SOLUTIONS

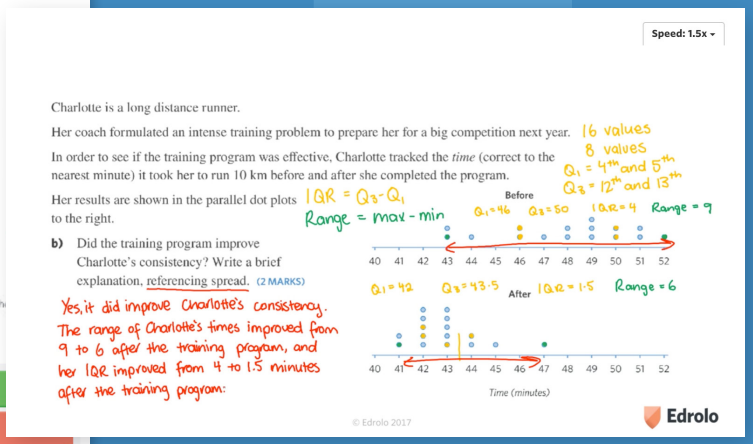


All questions found in the text are also available as interactive digital questions. To access these, click on the **X questions** button next to the corresponding theory lesson.

For multiple-choice questions, students receive immediate feedback.



Click the **Additional Resources** icon beside each **Area of Study** to find your **Textbook PDFs**.



CHECKING STUDENT RESPONSES

Multiple-choice results: see each student's answer and reflection on their understanding as well as a summary of your whole class.

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